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THE INFLUENCE OF PROPERTY VARIATION ON FORCED CONVECTION HEAT TRANSFER TO LIQUIDS

R. M. FAND and K. K. KESWANI
Department of Mechanical Engineering, University of Hawaii, Honolulu, Hawaii, U.S.A.

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Abstract—In this paper a new method of accounting for the influence of property variation on heat transfer by forced convection from heated surfaces to liquids is presented. The method is used in conjunction with new experimental data to develop a correlation equation for forced convection heat transfer from cylinders to liquids in crossflow. The method is also applied to heat transfer by forced convection to liquids from spheres and to turbulent flows of liquids in pipes.

NOMENCLATURE

a, b,	numerical constants;
C,	numerical constant;
c,	specific heat at constant pressure;
D,	cylinder or pipe diameter;
f, g,	denote functional relationships;
h,	heat-transfer coefficient;
i,	magnitude of exponent on viscosity ratio, see equations (4), (5) and (27)
j,	a number between zero and one, see equation (2);
k,	thermal conductivity;
L,	cylinder or pipe length;
m,	numerical constant;
N,	Nusselt number: $N = hDk^{-1}$;
n,	correction exponent for Nusselt number, see equation (7);
P,	Prandtl number: $P = c\mu k^{-1}$;
p,	correction exponent for Prandtl number, see equation (7);
Q,	electric power delivered to test section of heated cylinder;
q,	heat transfer per unit area per unit time from test section of heated cylinder;
R,	Reynolds number: $R = DU\rho\mu^{-1}$;
r,	correction exponent for Reynolds number, see equation (7);
T,	absolute temperature [deg R or deg K];
t,	temperature [deg F or deg C];
U,	fluid velocity;
x, y, z,	numerical constants.

Greek letters

β,	numerical constant;
γ,	ratio of specific heat at constant pressure to specific heat at constant volume;
μ,	dynamic viscosity;
ρ,	density.

Subscripts

b,	refers to bulk fluid conditions far from a heat transfer surface;
f,	refers to fluid conditions at the mean fluid temperature, see equation (2);
s,	refers to fluid conditions at a heat transfer surface.

INTRODUCTION AND STATEMENT OF OBJECTIVES

IT IS well known that for heat transfer by forced convection to perfect gases or incompressible fluids (liquids)* the Nusselt number, N, is functionally dependent upon the Reynolds number, R, and the Prandtl number, P. This statement may be expressed symbolically as follows:

$$f(N, R, P) = 0. \quad (1)$$

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* Viscous dissipation is here assumed to be negligible.

Equation (1) is derivable from the theory of dimensional analysis and also from the governing differential equations of continuity, momentum and energy. Equation (1) is ambiguous in the sense that it leaves unanswered the question as to what is an appropriate method for evaluating the *variable fluid properties* that enter into N, R and P. Two methods are presently employed to account for property variation. In one of these methods all properties are evaluated at the mean film temperature, t_f, defined as follows:

$$t_f = t_b + j(t_s - t_b); j = \tfrac{1}{2}. \tag{2}$$

With this approach equation (1) becomes

$$f(N_f, R_f, P_f) = 0 \tag{3}$$

which is no longer ambiguous relative to the question of property variation. However, it is intuitively clear that equation (3) can, at best, only approximately account for property variation. There is no *a priori* reason to believe that all properties should be evaluated at the *same* temperature, nor is there an *a priori* reason why $j = \frac{1}{2}$ should be adopted in equation (2) in preference to some other value; in fact, values of j different from $\frac{1}{2}$ have been sometimes recommended. The criterion as to which value of j should be adopted is: which value of j gives the least error when the resulting correlation equation is tested against experimental data. The mean film temperature method of accounting for fluid property variation is usually adequate when the fluid medium is a gas [1]. This method is less satisfactory when the fluid medium is a liquid, because the fluid properties of liquids (excluding density) generally are stronger and more complex functions of temperature than they are for gases. In order to better account for property variation in liquids, a second general method—different from the mean film temperature method—has been developed. Although this second method was originally developed to handle liquids, it is applicable to both liquids and gases.

The second method of accounting for property variation referred to above was proposed by Colburn [2] and the concept will be introduced here via the general form of an equation that is widely used to correlate heat transfer to fluids flowing in pipes:

$$N_b = CR_b^y P_b^z \left(\frac{\mu_b}{\mu_s}\right)^i. \tag{4}$$

Here all properties are evaluated at the bulk temperature, t_b, except in the viscosity correction factor, $(\mu_b/\mu_s)^i$. This correction factor is not derivable from the usual dimensional analysis nor from the governing differential equations; in other words, equation (4) is not a particular form of equation (1). The viscosity correction factor is a purely empirical quantity. Now, one might reasonably inquire why ratios of properties other than the viscosity, such as (k_b/k_s), do not also appear in equation (4). The answer is that such properties usually vary much less than does the viscosity. Nevertheless, there are important fluids for which such other properties do vary considerably; for example, the thermal conductivity of water at 50°F is 84 per cent of its value at 200°F. Thus, at least in the case of water, there is justification for including an empirical correction factor in equation (4) for the thermal conductivity.

The right hand side of equation (4) consists of a product of four factors including a numerical constant, the Reynolds and Prandtl numbers raised to different powers, and a viscosity correction factor. Since the Reynolds and Prandtl numbers both contain the viscosity, it is logical to regard the viscosity correction factor as a *net* correction for the separate viscosity variations in Reynolds and Prandtl numbers. However, this logic breaks down when the viscosity correction factor is applied, as it often is, to external flow. For example, Vliet and Leppert [3] have proposed the following form for correlating heat transfer from spheres to liquids:

$$N_b = (C_0 + C_1 R_b^y)\, P_b^z \left(\frac{\mu_b}{\mu_s}\right)^i \tag{5}$$

which may be written

$$N_b = C_0 P_b^z \left(\frac{\mu_b}{\mu_s}\right)^i + C_1 R_b^y P_b^z \left(\frac{\mu_b}{\mu_s}\right)^i. \qquad (6)$$

In equation (6) the viscosity correction factor can no longer be regarded as a *net* correction for the separate viscosity variations in the Reynolds and Prandtl numbers, because the right hand side of equation (6) contains a *sum* of two products both of which contain the same viscosity correction factor but only one of which contains the Reynolds number.

In spite of the conceptual difficulties discussed above, the correction factor method of accounting for property variation possesses certain advantages over the mean film temperature method. There is, first, an advantage with respect to the performance of certain kinds of calculations [4, 5]. Another, more important, advantage of the correction factor method is that it has the potential to represent experimental data (reality) with greater accuracy than does the mean film temperature method. That this should be so is clear when one recalls that the solutions to the governing set of differential equations, however complex they may be, must be a function of the boundary temperatures. The preceding observation leads one to inquire whether it is possible to devise a scheme of accounting for property variation which will possess the advantages of the viscosity correction ratio method while avoiding the theoretical and conceptual difficulties described above. Such a scheme has been developed and is described in what follows.

The method proposed here to account for property variation is simple. The basic idea is to replace equation (3) with the following equation:

$$f(N_{scn}, R_{scr}, P_{scp}) = 0 \qquad (7)$$

where $N_{scn} = N_s(N_b/N_s)^n$, $R_{scr} = R_s(R_b/R_s)^r$ and $P_{scp} = P_s(P_b/P_s)^p$.

The quantity N_{scn} is to be read "surface Nusselt number with correction factor exponent n," and similarly for the Reynolds and Prandtl numbers. With this notation $N_{scn=1} = N_b$, and similarly for the Reynolds and Prandtl numbers.

Equation (7) implies that in all forced convection correlation equations, whatever their particular form may be, each of the three dimensionless parameters appearing therein must be corrected for property variation *where it stands in the equation* by means of a multiplying correction factor. This equation is consistent with the governing set of differential equations and with the theory of dimensionless analysis; furthermore, its use eliminates the problem of "net corrections" discussed above in relation to equations (4) and (6). Equation (7) forms the basis of the method of accounting for property variation developed herein.

The objectives of the present work were three in number: first, to obtain accurate experimental data on forced convection heat transfer from cylinders to water in crossflow in a region of flow for which such data are not presently available (Reynolds number from approximately 400 to 3700); second, to use this data, together with previously published information, to quantify the method of accounting for fluid property variation proposed in equation (7) for cylinders exposed to crossflows of liquids; third, to expand the field of application of this method to geometries other than cylinders in crossflow—in particular to spheres in forced flow and to turbulent pipe flow.

REVIEW OF THE LITERATURE

Cylinders in crossflow

In 1924, Davis [6] obtained heat-transfer data for cylinders subjected to crossflows of distilled water, paraffin and three different kinds of transformer oil. His experimental apparatus consisted of a circular trough containing liquid and a whirling cantilever arm which moved electrically heated wires through the trough. In these experiments the Reynolds number ranged from approximately 0·14 to 170. McAdams [7] analyzed these data and determined that the following equation represents Davis' data within 15 per cent:

$$N_f = 0{\cdot}86\, R_f^{0{\cdot}43}\, P_f^{0{\cdot}3}. \tag{8}$$

Ulsamer [8] subsequently correlated Davis' data for liquids and those of several investigators on air by the following equation:

$$N_f = CR_f^m P_f^{0{\cdot}31} \tag{9}$$

where the values of C and m are, respectively, 0·91 and 0·385 for $0{\cdot}1 < R_f < 50$, and 0·6 and 0·5 for $50 < R_f < 10000$. Equation (9) contains a discontinuity at $R_f = 50$ which does not exist in reality. Kramers [9] devised a formula, based upon the same data referred to in Ulsamer's paper, which contains no such discontinuity:

$$N_f = 0{\cdot}42\, P_f^{0{\cdot}20} + 0{\cdot}57\, R_f^{0{\cdot}50} p_f^{0{\cdot}33}. \tag{10}$$

King [10] performed a theoretical calculation of heat transfer from cylinders to air in crossflow in which he obtained the following relationship:

$$N = \frac{1}{\pi} + \sqrt{\left(\frac{2}{\gamma\pi} RP\right)}. \tag{11}$$

Piret *et al.* [11] mounted fine electrically heated wires across a nozzle discharging water and obtained data at low Reynolds numbers (approximately 0·08–16). They determined the following correlation equation:

$$N_f = 0{\cdot}965\, R_f^{0{\cdot}28}\, P_f^{0{\cdot}30}. \tag{12}$$

It is important to note that the exponent of the Reynolds number in equation (12) is substantially lower than in equation (8); this means that the *trend* of Piret's data differs from that of Davis. McAdams [5] analyzed the data of Davis and of Piret and determined the following relation:

$$N_f = (0{\cdot}35 + 0{\cdot}56\, R_f^{0{\cdot}52})\, P_f^{0{\cdot}3}. \tag{13}$$

Equation (13) is at present the most widely used correlation equation for heat transfer by forced convection from cylinders to liquids in crossflow.

In 1965 Fand [12] performed a series of six heat-transfer tests with a cylinder placed in a crossflow of water. In these tests the Reynolds number varied from approximately 10^4 to 6×10^4. McAdams' equation predicts the results of these tests within 3 per cent, which is remarkable in view of the limited range of the data to which McAdams had access originally (approximately $0{\cdot}1 < R < 170$). However, there is still a broad gap in the published literature ($170 < R < 10^4$) for which no experimental data are available to thoroughly check the validity of equation (13). As has been stated earlier, one of the objectives of the present work was to obtain accurate experimental data in this unexplored range.

In recent years, efforts have been made to represent heat transfer from cylinders in crossflow by equations having the following form:

$$N = g\{aR^{\frac{1}{2}} + bR^{\beta}\}\, P^{z}. \tag{14}$$

In this formulation, the term containing $R^{\frac{1}{2}}$ represents heat transfer through the laminar boundary layer on the front portion of the cylinder, and the term containing R^{β} accounts for the contribution to the total heat transfer from the rear portion of the cylinder, where separation occurs. Two examples of such correlations [13, 12] are:

$$N_b\left(\frac{\mu_s}{\mu_b}\right)^{0{\cdot}25} = [0{\cdot}30\, R_b^{0{\cdot}50} + 0{\cdot}10\, R_b^{0{\cdot}67}]\, P_b^{0{\cdot}40}, \tag{15}$$

$$N_f = [0{\cdot}35 + 0{\cdot}34\, R_f^{0{\cdot}50} + 0{\cdot}15\, R_f^{0{\cdot}58}]\, P_f^{0{\cdot}30}. \tag{16}$$

Spheres in forced flow

In 1946 Kramers [9] conducted an experimental investigation of heat transfer from spheres to air, water and oil. On the basis of these experiments, Kramers developed the following correlation equation:

$$N_f = 2{\cdot}0 + 1{\cdot}3\, P_f^{0{\cdot}15} + 0{\cdot}66\, R_f^{0{\cdot}50}\, P_f^{0{\cdot}3}. \tag{17}$$

Vliet and Leppert [3], using the same basic experimental technique as Kramers, have determined the following empirical correlation equation for water:

$$N_b = (2{\cdot}7 + 0{\cdot}12\, R_b^{0{\cdot}66})\, P_b^{0{\cdot}5}\left(\frac{\mu_b}{\mu_s}\right)^{0{\cdot}25}. \tag{18}$$

On the basis of a comparison of their experimental results with those of Kramers and others, Vliet and Leppert deduced the following relation for heat transfer from spheres to fluids of Prandtl number from 2 to 380:

$$N_b = (1{\cdot}2 + 0{\cdot}53\, R_b^{0{\cdot}54})\, P_b^{0{\cdot}3} \left(\frac{\mu_b}{\mu_s}\right)^{0{\cdot}25}. \qquad (19)$$

For heat transfer from spheres to liquids in the range of Reynolds number from 1 to 200, Kreith [14] recommends the following correlation:

$$N_f = \left[0{\cdot}97 + 0{\cdot}68 \left(\frac{DU\rho_b}{\mu_f}\right)^{0{\cdot}5}\right] P_f^{0{\cdot}3}. \qquad (20)$$

For constant-density fluids, equation (20) may be written:

$$N_f = (0{\cdot}97 + 0{\cdot}68\, R_f^{0{\cdot}5})\, P_f^{0{\cdot}3}. \qquad (21)$$

Flow in pipes

The literature on heat transfer to fluids flowing in pipes is voluminous, because of the great importance of this geometry. Only a few of the many papers that have been published on this subject are briefly reviewed here.

In 1928 Morris and Whitman [15] obtained heat-transfer data for three petroleum oils and for water flowing through the inner tube of a horizontal double-pipe heat exchanger. The experiments covered both heating and cooling for Reynolds numbers between approximately 600 and 45000. Morris and Whitman presented their results graphically since they could not find a simple equation which would represent their entire body of data. In their presentation the properties of fluids were evaluated at t_b.

Dittus and Boelter [16] studied the data of Morris and Whitman on heating and cooling oil and also the results of several investigations on air and water, and based thereon proposed the following equations:

$$N_b = 0{\cdot}0243\, R_b^{0{\cdot}8}\, P_b^{0{\cdot}4} \quad \text{(for heating)} \qquad (22)$$

$$N_b = 0{\cdot}0265\, R_b^{0{\cdot}8}\, P_b^{0{\cdot}3} \quad \text{(for cooling).} \qquad (23)$$

In 1932 Sherwood and Petrie [17] published extensive data on heat transfer to several liquids (water, acetone, benzene, kerosene and n-butyl alcohol) for both laminar and turbulent flow through a steam-jacketed copper pipe. They found that the data for turbulent flow were well correlated by the Dittus–Boelter equation. In 1933 Colburn [2] studied the results of several investigations and proposed a correlation equation for turbulent flow in pipes which may be written in the following form:

$$N_b = 0{\cdot}023\, R_b^{0{\cdot}8}\, P_b^{\frac{1}{3}} \left(\frac{\mu_b}{\mu_f}\right)^{7/15}. \qquad (24)$$

Sieder and Tate [18] analyzed various data on the heating and cooling of liquids flowing in pipes. They presented their results in the form of a graph in which log $[N_b P_b^{-\frac{1}{3}} (\mu_b/\mu_s)^{-0{\cdot}14}]$ was plotted against log (R_b) for various values of L/D. The following equation represents Sieder and Tate's graphical presentation in the region of fully developed turbulent flow:

$$N_b = 0{\cdot}027\, R_b^{0{\cdot}8}\, P_b^{\frac{1}{3}} \left(\frac{\mu_b}{\mu_s}\right)^{0{\cdot}14}. \qquad (25)$$

Kreith and Summerfield [19] have performed experiments on heat transfer to liquids (water, aniline and n-butyl alcohol) flowing through an electrically heated tube. The range of the Reynolds number was from 36000 to 78000. On the basis of their experimental data, Kreith and Summerfield concluded that the exponent on the viscosity ratio in equation (25) is too high. They proposed the following correlation equation:

$$N_b = 0{\cdot}034\, R_b^{0{\cdot}8}\, P_b^{\frac{1}{3}} \left(\frac{\mu_b}{\mu_s}\right)^{0{\cdot}10}. \qquad (26)$$

More recently, Malina and Sparrow [20] have concluded on the basis of experiments that the exponent of the viscosity ratio for heating liquids flowing turbulently in pipes should be 0·05.

Petukhov [21] has carried out an extensive mathematical analysis of the problem of heat transfer in turbulent pipe flow with variable physical properties. His complete analytical solution is rather complicated and requires the numerical solution of certain integral equations with the aid of a computer. The procedure is considerably simplified in the case of constant physical properties. Petukhov compared his constant properties solution with the experimental results of various investigators by extrapolating their data to zero temperature difference and found good agreement. Then, in order to account for the effect of property variation without solving the complex integral equations he had derived, Petukhov proposed that a viscosity correction factor be applied to his constant properties solution in the following manner:

$$N_b = N_{0b}\left(\frac{\mu_b}{\mu_s}\right)^i \tag{27}$$

where $i = 0{\cdot}11$ and $0{\cdot}25$ for heating and cooling of liquids, respectively, and N_{0b} is Petukhov's solution for the case of constant physical properties given by

$$N_{0b} = \frac{(\xi/8)\,R_b P_b}{K_1 + K_2(\xi/8)^{\frac{1}{2}}\,(P_b^{\frac{2}{3}} - 1)}$$

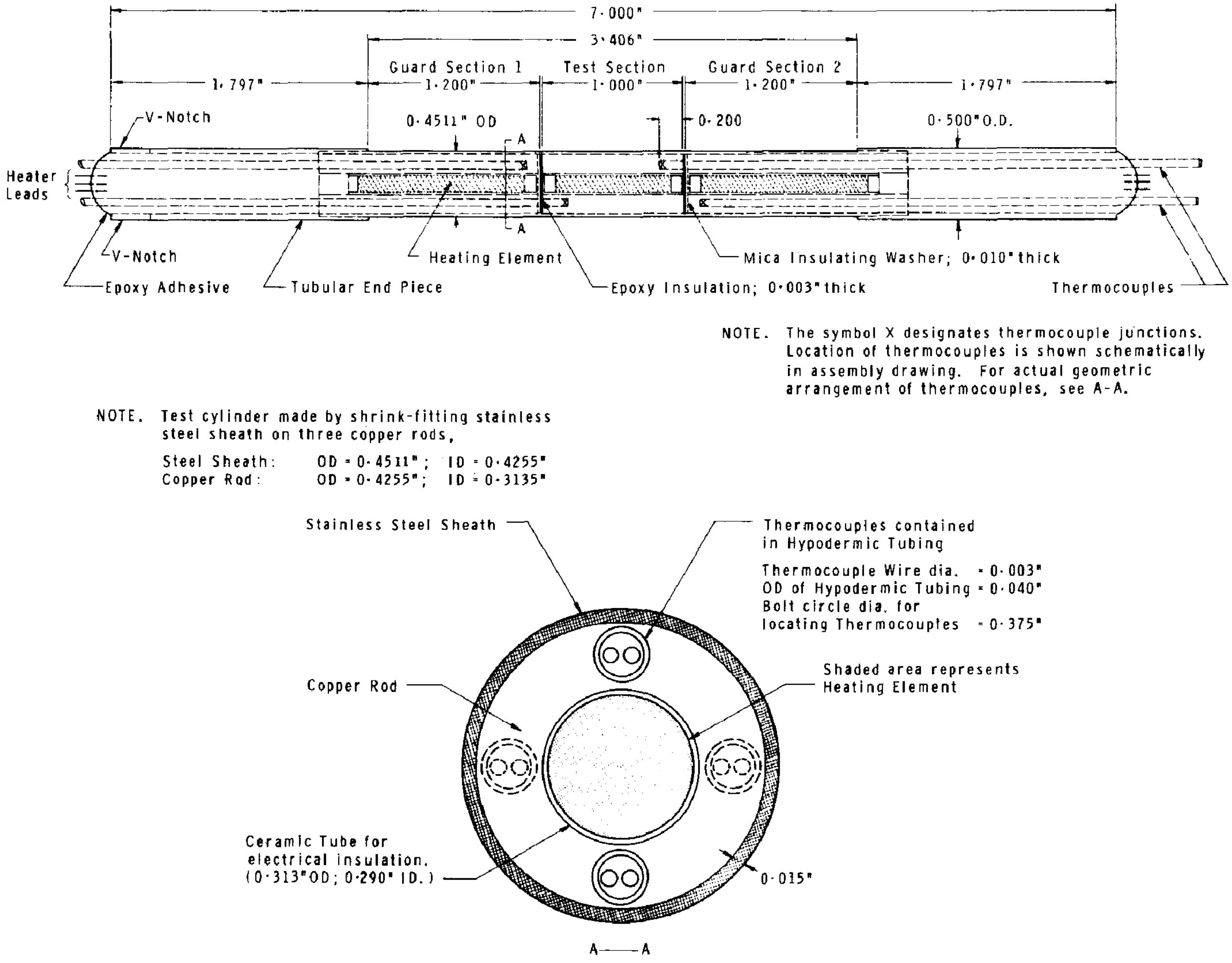
where $\xi = (1{\cdot}82 \log R_b - 1{\cdot}64)^{-2}, K_1 = 1 + 3{\cdot}4\xi$ and $K_2 = 11{\cdot}7 + 1{\cdot}8\,P_b^{-\frac{1}{3}}$. Petukhov states that equation (27) is valid for values of (μ_b/μ_s) between 0·025 and 12·5, R_b between 10^4 and $1{\cdot}25 \times 10^5$, and P_b between 2 and 140.

APPARATUS AND EXPERIMENTAL PROCEDURE

The experimental portion of the present investigation was conducted by placing a cylindrical, electrically heated test specimen horizontally in a water tunnel, such that the flow of water was horizontal and normal to the axis of the test specimen (crossflow). Simultaneous measurements of velocity, power input to the test specimen, surface temperature and bulk water temperature were made, from which the heat-transfer coefficient and all other pertinent parameters could be calculated. The ranges of the experimental variables were: $q = 10302$–77046 Btu/hft^2, $U = 0{\cdot}12$–$0{\cdot}43$ ft/s, $D = 0{\cdot}4511$ in., $t_b = 50, 70, 100, 140$ deg F and $\Delta t = 18{\cdot}3$–$156{\cdot}4$ deg F.

The water tunnel used here is described in detail in [22]. This tunnel consists basically of a closed loop through which water can be pumped at controlled and measured values of velocity, temperature and pressure. The bulk water temperature in the tunnel was measured by means of a copper–constantan thermocouple located upstream from the test specimen.

The test specimen used in these experiments is described in [23] and is shown in Fig. 1. It consists basically of three cylindrical sections laid end to end, which are thermally insulated from one another. The central "test section" (length, L = 1·000 in.) is flanked by two "guard sections". Each section can be heated independently by feeding power through three variable autotransformers to ribbon heating elements contained therein. The diameter, D, of the heated sections is 0·4511 in. The surface temperature of the central test section of the cylinder is measured by two internal copper–constantan thermocouples located 90° to each other, relative to the axis of the test cylinder. Each guard section contains one thermocouple. During experiments the temperatures of the guard sections are adjusted to be equal to the average temperature of the test section thermocouples, by feeding appropriate power into the guard heaters. Under these conditions there is negligible transfer of heat from the central test section by axial conduction. Since radiation from the surface of the tests specimen was calculated and found to be negligible (less than 0·15 per cent of the total energy transfer), it follows that substantially all of the electrical power fed into the test section was dissipated by convection from its surface to the surrounding water. Since the thermocouples contained in the test section are embedded below its surface, the temperature recorded by them is not exactly equal to the surface temperature of the test section. In order to determine the surface temperature, the radial temperature drop between the geometric centers of the thermocouples and the surface of the test cylinder was



NOTE. The symbol X designates thermocouple junctions. Location of thermocouples is shown schematically in assembly drawing. For actual geometric arrangement of thermocouples, see A-A.

NOTE. Test cylinder made by shrink-fitting stainless steel sheath on three copper rods,

Steel Sheath: OD = 0·4511"; ID = 0·4255"
Copper Rod: OD = 0·4255"; ID = 0·3135"

Thermocouple Wire dia. = 0·003"
OD of Hypodermic Tubing = 0·040"
Bolt circle dia. for locating Thermocouples = 0·375"

FIG. 1. Drawing of test specimen.

calculated by assuming that the simple conduction equation for radial heat transfer through composite cylinders is applicable. The electric power, Q, fed into the test section of the heated cylinder was measured with an error of less than 0·1 per cent by means of a carefully calibrated wattmeter. A precision potentiometer was used to read the output of all thermocouples with an accuracy of better than 0·05 deg F.

The water tunnel was calibrated with respect to velocity by means of the "hydrogen bubble technique". In the hydrogen bubble technique for measuring the velocity of water, minute bubbles of hydrogen are produced in the stream by dissociation. These bubbles move along with the flow and the velocity of flow is determined by observing the motion of the bubbles. In the present study, hydrogen bubbles were generated on a fine Nichrome wire (0·001 in. dia.) held vertically in the water tunnel. The wire was incorporated into the electro-mechanical apparatus shown schematically in Fig. 2. With this apparatus, electric power was fed intermittently to the wire through a variable speed motor-driven rotating mechanical switch (rotating commutator). The principle of operation of the rotating commutator switch may be understood by studying Fig. 3. As the commutator rotates, the two carbon brushes alternately contact two separate metal surfaces and then the same metal surface, thus supplying intermittent bursts of power to the wire, and thereby producing intermittent sheets of bubbles. The angle, θ, between the commutator brushes could be varied and the brushes could be locked into position at any angle θ by means of a spring-loaded cam mechanism not shown in Fig. 3. Thus, for one revolution of the commutator, the duration of the electric current flowing through the bubble generating wire could be varied from zero to a time approximately equal to the time of half a revolution. In this manner, the width of the bubble-sheets flowing downstream from the wire could be varied. In the actual experiments, the width of the bubble sheets was adjusted to be as small as possible, so that, ultimately, fine lines of bubbles were produced. Photographs of such lines are shown in Fig. 4. The bubble lines are closer or wider apart, depending on the velocity of flow and the rotational speed of the motor-driven commutator. Two different spacings of bubble-lines are shown in Fig. 4 in order to demonstrate the capability of this equipment. The shapes of the bubble-lines (see

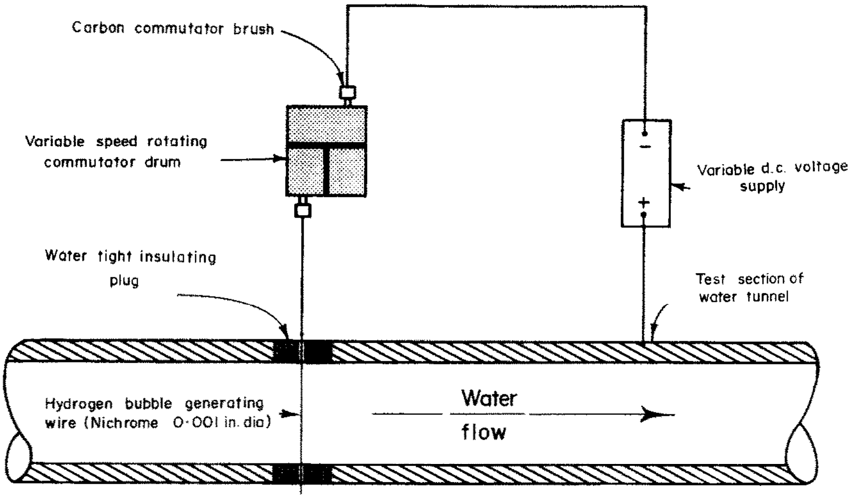


FIG. 2. Electro-mechanical bubble generating apparatus.

Fig. 4) indicate that the velocity profiles in the central region of the tunnel were very nearly flat in the entire range of velocities measured. The coherent motion of the bubbles also demonstrates that no free stream turbulence was present in the experiments conducted here.

To determine the water velocity in the tunnel it was necessary to measure the distance between two successive bubble fronts. In order to measure the distance between two lines of bubbles, a stroboscope was placed behind a glass window inserted into the tunnel. A microscope was focussed on the center of the bubble-generating wire through another glass window. The frequency of the stroboscope light was varied until the lines of bubbles appeared stationary. Under these conditions the frequency of the stroboscope light was equal to the frequency with which the bubble lines were being generated by the wire. A calibrated eye-piece was used with the microscope to measure the distance between two consecutive bubble lines. The frequency of the stroboscope was measured by an electronic counter, thereby providing accurate information on the interval of time elapsed between two consecutive flashes of the stroboscope light. The water velocity is equal, of course, to the distance between bubble fronts measured with the microscope divided by the time interval measured by the electronic counter.

Average or "bulk" velocity measurements were made by weighing the quantity of water discharged from the test section of the tunnel during prescribed intervals of time. In the bulk velocity measurements, the time of discharge was measured with an accuracy of 0·3 per cent and the weight of water was measured with an accuracy of 0·05 per cent. It was observed that at velocities higher than 0·1 ft/s, there was no discernible difference between the centerline velocity (as measured by the bubble technique) and bulk velocity. This indicated that at velocities higher than 0·1 ft/s, the velocity profile over the entire tunnel cross section was nearly flat.

Centerline and bulk velocity data were related to the pressure drop across orifices incorporated into the tunnel circuit. A 0·2 in. dia. orifice was used for velocities less than 0·15 ft/s, and a 0·4 in. dia. orifice was used for velocities greater than 0·15 ft/s. The pressure drop across the orifices was measured with a mercury manometer when the pressure drop exceeded 1 psia; at low velocities, when the pressure drop across the orifices was less than 1 psia, an inverted inclined (water) manometer was used. It is estimated that the velocity calibration achieved by means

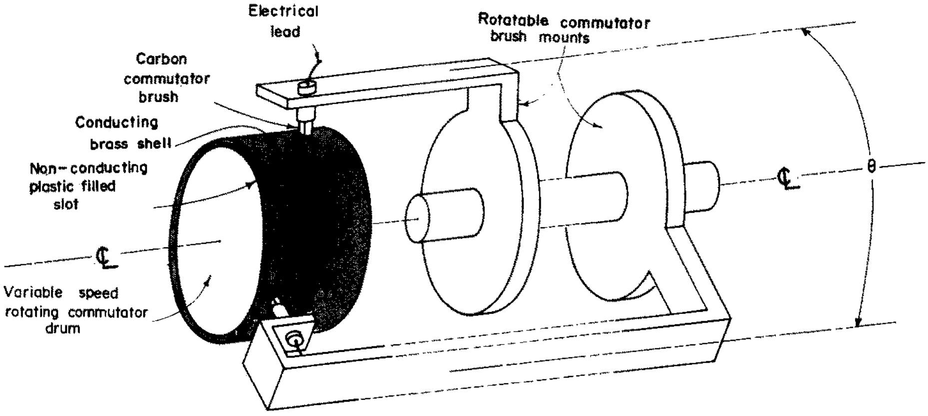


FIG. 3. Rotating commutator switch.

of the techniques described above has an accuracy of better than 1 per cent over the velocity range employed here (0·12–0·43 ft/s).

When a test specimen is inserted into the water tunnel, it obstructs the flow and hence necessitates a correction for blockage. Three methods for making this correction are described in the literature. The method of Vliet and Leppert [3] was adopted here. In this method the correct velocity, U, is obtained by applying the continuity equation as follows:

$$U = \left[\frac{A_{\text{unobstructed}}}{A_m}\right] U_{\text{unobstructed}}$$

where the mean area, A_m, is defined as the area which, when multiplied by the test specimen diameter, is equal to the net volume of the fluid at the location of the cylinder. For the geometry of the present investigation,

$$U = 1{\cdot}1525\, U_{\text{unobstructed}}. \qquad (28)$$

One of the difficulties encountered in the performance of heat-transfer experiments in water is the adhesion of air bubbles to the heat transfer surface. Piret [11] speaks of this problem. It was largely avoided during the present study by taking the following measures: first, the water was heated prior to its use (160deg F for 4h) in order to drive off dissolved air; second, the water was "aged" for at least two days prior to use—this allows microscopic bubbles to dissolve, which might otherwise serve as nucleation centers; third, the tunnel was pressurized (approximately 70 psia) in order to inhibit the formation and reduce the size of bubbles—this pressurization also prevented boiling. The test specimen was observed visually through a window located above it. Test data were taken only when the visible portion of the cylinder was completely devoid of air bubbles.

The heat-transfer coefficient for a cylinder in crossflow varies along the circumference of the cylinder. Therefore, the temperature of a thermocouple embedded near the surface of such a cylinder will be a function of the angular position of the termocouple relative to the direction of fluid flow. In the present investigation the measured difference between t_s and t_b was found to vary by as much as 15 per cent, depending upon the angular orientation of the test cylinder. In order to cope with this variation, the test specimen was rotated in 15 degree increments of arc (all else held constant) and the temperature difference was measured at each 15 degree interval until the cylinder had been turned through 360 degrees (24 readings). These readings were plotted and the integrated mean value of the temperature difference, designated Δt, was used to calculate the heat-transfer coefficient, h, reported herein. The determination of a single value of Δt by the method just described involves 24 measurements. The time required to make such a determination would have been prohibitive if it had been necessary to apply it directly to every data point. Fortunately, it was not necessary to perform the rotation procedure for every data point. The majority of data points could be obtained without rotation because of the following two experimentally verified facts: (1) the maximum difference between t_s and t_b as the cylinder is rotated, designated $\Delta t_{\max}$, always occurred when the heated test cylinder was located in one and the same angular position relative to the direction of fluid flow; (2) the ratio $\Delta t/\Delta t_{\max}$ was nearly a constant ($0{\cdot}93 \pm 0{\cdot}02$) for all test conditions. Thus, once having experimentively determined the magnitude of the ratio $\Delta t/\Delta t^{*}_{\max}$, it was possible to leave the cylinder in the position corresponding to $\Delta t_{\max}$ and to take *one* temperature reading, i.e. $\Delta t_{\max}$, from which Δt could then be readily calculated. It is conservatively estimated that the experimental error involved in the determination of h by the methods described above does not exceed 3 per cent.

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* The value adopted, 0·93, is the average of seven values obtained by the rotation procedure at different conditions covering the entire range of the experimental parameters.

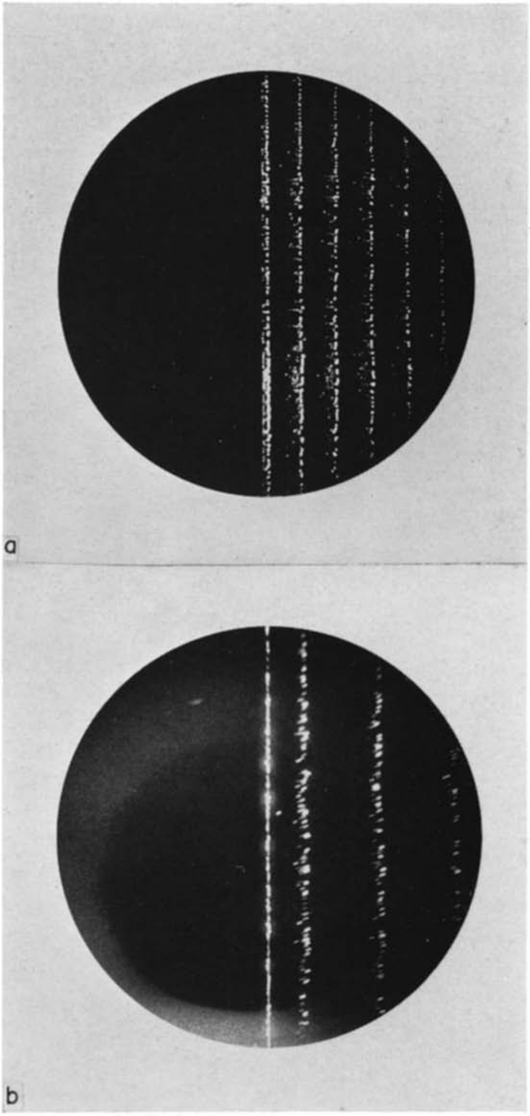
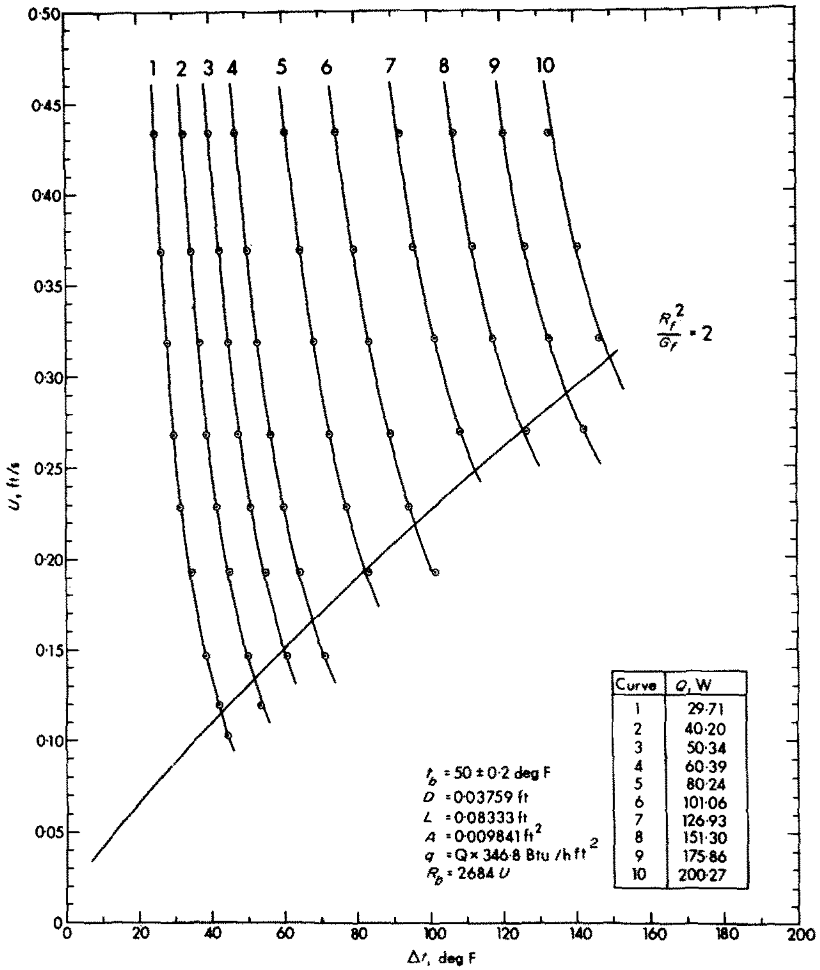


FIG. 4. Photographs of hydrogen bubble lines.

PRESENTATION OF DATA

Four sets of experimental data were taken, at t_b = 50, 70, 100 and 140 deg F. In each set the bulk temperature was maintained constant to within 0·2 deg F. The data points were plotted as shown in Figs. 5–8 and curves were then fared through these points. Crossplots of the fared curves were made (not presented here) in order to ascertain that the fared curves were self-consistent As may be seen in the figures, most of the data points lie very nearly on the fared curves. The fared curves shown (greatly reduced in size) in Figs. 5–8 were used as the basis for calculating the parameters of interest in this study, because these curves are believed to represent the overall experimental results accurately. By regarding the curves as the data, it is possible to determine the Nusselt number at precisely equal increments of the Reynolds number (see Table 1); and this, in turn, results in equal "weighting" of all regions of the data in the correlation analysis which follows.



Curve	Q, W
1	29·71
2	40·20
3	50·34
4	60·39
5	80·24
6	101·06
7	126·93
8	151·30
9	175·86
10	200·27

FIG. 5. Experimental data at t_b = 50 deg F.

CORRELATION OF DATA FOR CYLINDERS IN CROSSFLOW

General comments

The experimental results presented in Figs. 5–8 were correlated on the basis of equation (3) (mean film temperature correlation) and also on the basis of equation (7) (correction exponent correlation). This was done by reading off values of the Nusselt numbers from the curves at equal intervals of Reynolds number as shown, by way of example, in Table 1. All the data used lie in the region above the lines correponding to $R_f^2/G_f > 2$ in Figs. 5–8, which insures that natural convection effects are negligible [14]. Physical properties of water were calculated from formulae given in [25–27].

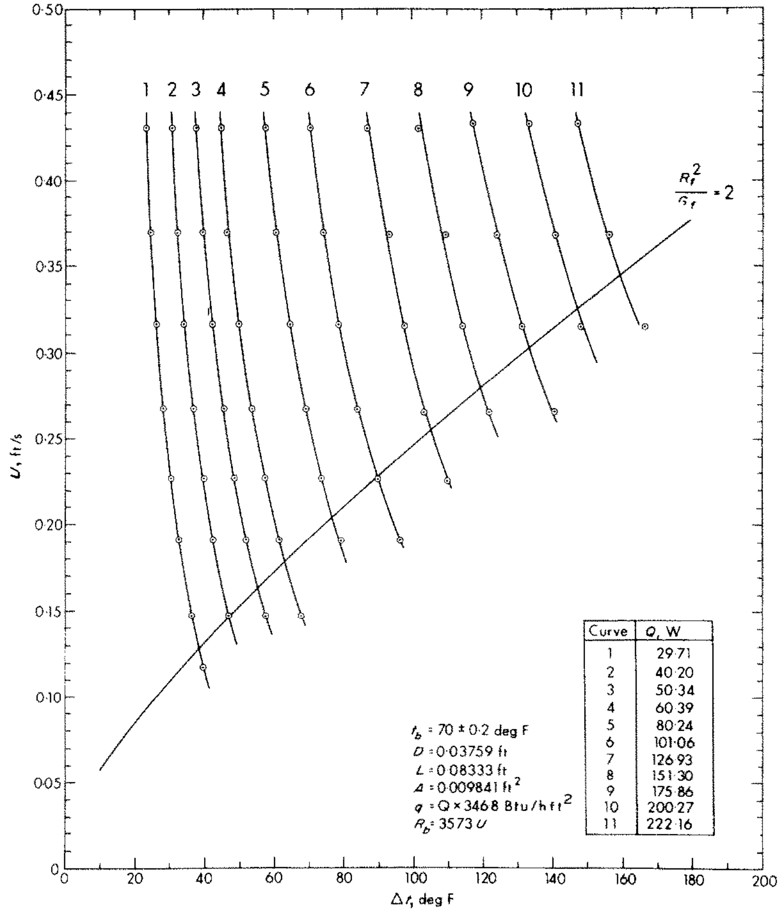


FIG. 6. Experimental data at t_b = 70 deg F.

Correlation was performed with the aid of a computer, based upon the following hypothesis:

$$N = C_0 + C_1 P^{x_3} + (C_2 R^{\frac{1}{2}} + C_3 R^{x_1}) P^{x_2}. \quad (29)$$

Equation (29) is a composite of all the published correlation equations for cylinders presented in the review of the literature. In this equation, the symbol N stands for either N_f or N_{scn}, R stands for either R_f or R_{scr}, and P stands for either P_f or P_{scp}. The quantities x_1, x_2, x_3, n, r, p and the C's are all constants to be determined so as to best represent the present data and also the data of Davis [6], Piret [11] and Fand [12].

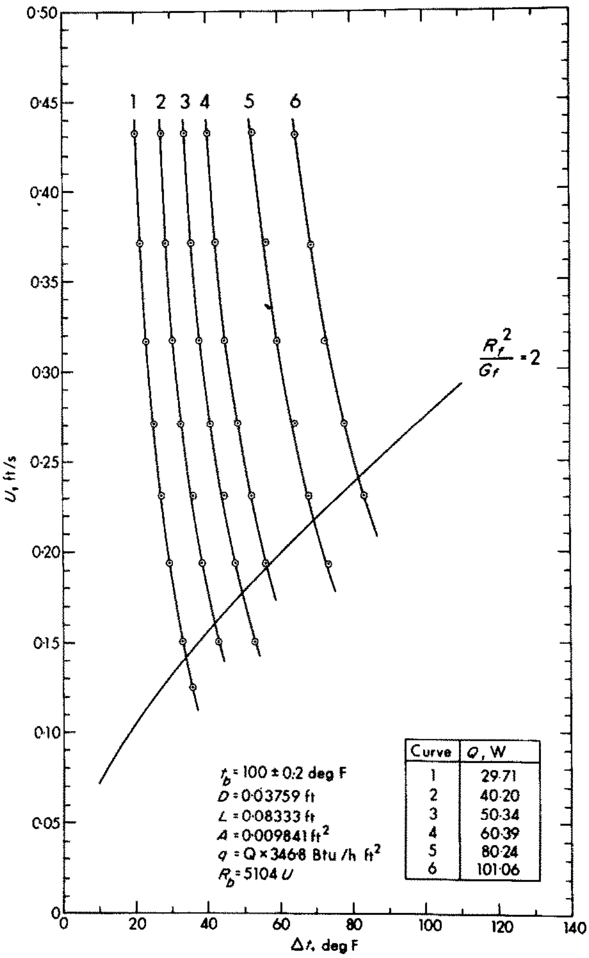


FIG. 7. Experimental data at t_b = 100 deg F.

In order to obtain a general idea of the relationship between the data obtained in the present study and previously published data, the present data as well as the data of Davis, Piret and Fand were compared with the predictions of McAdams' equation*. The mean error and standard deviation of the error of McAdams' equation relative to Davis' water data are 9·49 per cent and 4·72 per cent, respectively. The mean errors and standard deviations of the errors of McAdams' equation relative to other sets of data are listed in Table 2.

It is assumed here that the relatively large discrepancy (9·49 per cent) between McAdams' equation and Davis' water data is a result of the formation of air bubbles on the heated wires used in Davis' water experiments. This assumption is based upon the fact that in Davis' experiments the pressure was atmospheric, and the

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* The data were also compared with equations (15) and (16). These two equations were found to be in greater error relative to the data than McAdams' equation (see Table 2).

apparatus did not allow visual observation of the heated specimens during tests; it is also based on Piret's published comments concerning bubble formation and upon the experience of the present investigators. In view of the doubtful validity of Davis' water data, these water data have not been used here for correlation purposes; however, Davis' oil data form an important cornerstone of the present analysis, for reasons that are explained in what follows.

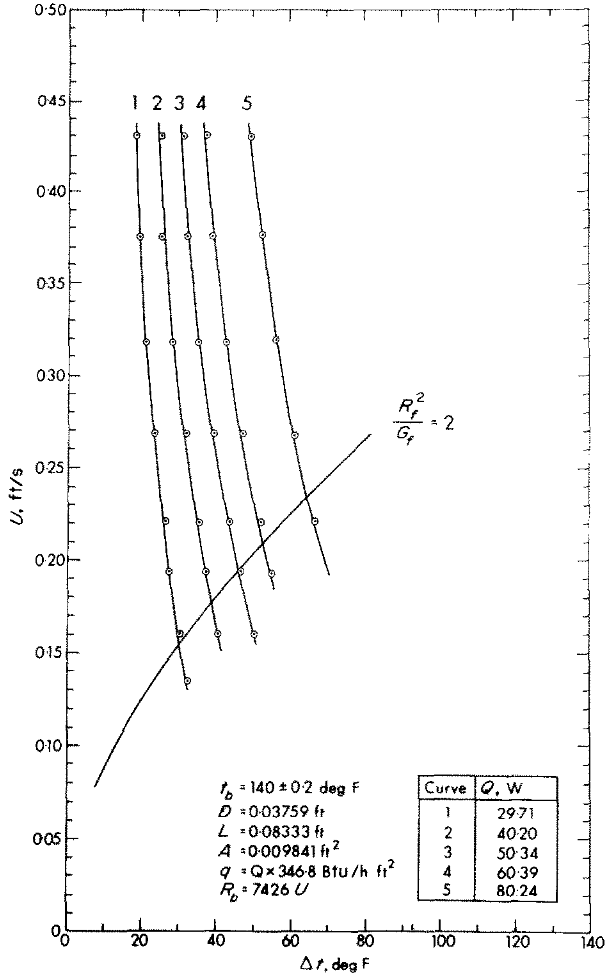

FIG. 8. Experimental data at t_b = 140 deg F.

The right-hand side of equation (29) consists of two terms which contain R and two terms which do not contain R. When the magnitude of R is 100 or more, the contribution of those terms which contain R predominate over those which do not. Therefore, it is possible to determine accurately the values of the coefficients and exponents of R in equation (29) from experimental data for which $R > 100$ by roughly approximating, or even totally neglecting, the terms not containing R. For this reason, the experimental data gathered during the present study ($400 < R < 3700$) could be used to determine the coefficients and exponents of the

Table 1. *Tabulation of values of* R_f, N_f and P_f *obtained from Fig.* 5

R_f	N_f	P_f	R_f	N_f	P_f	R_f	N_f	P_f
500	27·9	6·76	1300	43·6	7·52	1800	47·7	5·21
600	30·6	6·94	1300	43·4	7·02	1800	46·6	4·58
600	30·6	6·33	1300	43·1	6·58	1800	45·8	4·08
700	33·1	7·08	1300	42·6	6·17	1800	45·0	3·65
700	33·0	6·49	1300	41·9	5·46	1900	48·9	5·28
700	32·7	5·98	1300	41·0	4·83	1900	47·7	4·64
800	35·3	7·19	1400	45·0	7·57	1900	47·0	4·14
800	35·2	6·62	1400	45·0	7·09	1900	46·3	3·73
800	34·7	6·12	1400	44·6	6·65	2000	50·1	5·34
800	34·4	5·67	1400	44·2	6·25	2000	48·7	4·69
900	36·9	7·27	1400	43·2	5·54	2000	48·1	4·20
900	37·1	6·73	1400	42·4	4·92	2000	47·6	3·79
900	36·7	6·24	1400	41·0	4·24	2100	49·7	4·74
900	36·3	5·80	1500	45·0	7·57	2100	49·1	4·26
900	35·3	5·03	1500	46·6	7·15	2100	48·7	3·85
1000	38·6	7·34	1500	46·3	6·72	2100	47·9	3·49
1000	38·9	6·82	1500	45·7	6·32	2200	50·6	4·79
1000	38·5	6·34	1500	44·6	5·61	2200	50·0	4·30
1000	38·2	5·92	1500	43·9	5·00	2200	49·7	3·90
1000	37·2	5·17	1500	42·6	4·34	2200	49·0	3·54
1100	40·3	7·41	1600	48·2	6·80	2300	50·8	4·34
1100	40·4	6·89	1600	47·1	6·38	2300	50·5	3·94
1100	40·0	6·42	1600	46·0	5·69	2300	50·0	3·59
1100	39·8	6·01	1600	45·1	5·07	2400	51·6	4·38
1100	38·8	5·28	1600	44·1	4·43	2400	51·3	3·98
1100	37·8	4·62	1600	43·0	3·92	2400	50·9	3·63
1200	42·1	7·47	1700	48·5	6·44	2500	51·9	4·01
1200	41·9	6·96	1700	47·4	5·76	2500	51·6	3·67
1200	41·5	6·50	1700	46·4	5·15	2600	52·3	3·71
1200	41·3	6·09	1700	45·4	4·51	2700	53·0	3·74
1200	40·4	5·37	1700	44·5	4·00			
1200	39·5	4·73	1800	48·8	5·83			

Table 2. *Errors incurred by different correlation equations for cylinders in crossflow when tested against four sets of experimental data*

Equation	Present data		Davis oil data [6]		Fand data [12]		Piret data [11]	
	Mean error, %	Standard deviation of error, %	Mean error, %	Standard deviation of error, %	Mean error, %	Standard deviation of error, %	Mean error, %	Standard deviation of error, %
Equation (13): $N_f = (0{\cdot}35 + 0{\cdot}56\,R_f{}^{0{\cdot}52})\,P_f^{0{\cdot}3}$	− 5·28	2·08	− 2·45	10·13	1·35	1·76	− 2·04	7·05
Equation (15): $N_b = (0{\cdot}30\,R_b^{0{\cdot}50} + 0{\cdot}10\,R_b^{0{\cdot}67})\,P_b^{0{\cdot}4}\,(\mu_b/\mu_s)^{0{\cdot}25}$	15·20	6·28	7·02	14·84	54·53	8.53	−47·85	16·87
Equation (16): $N_f = (0{\cdot}35 + 0{\cdot}34\,R_f^{0{\cdot}5} + 0{\cdot}15\,R_f^{0{\cdot}58})\,P_f^{0{\cdot}3}$	−10·59	2·18	−10·75	9·94	0·48	1·72	− 9·23	5·82
Equation (32): $N_f = (0{\cdot}255 + 0{\cdot}699\,R_f^{0{\cdot}50})\,P_f^{0{\cdot}29}$	− 0·04	2·01	− 0·04	9·00	1·11	2·60	0·11	11·50
Equation (34): $N_{scn=0{\cdot}50} = (0{\cdot}221 + 0{\cdot}693\,R^{0{\cdot}50}_{scr=0{\cdot}75})\,P_b^{0{\cdot}29}$	0·04	1·94	− 0·01	7·19	0·46	2·61	− 0·73	11·69

terms containing R in equation (29). In order to determine the coefficients and exponents of the terms *not* containing R in equation (29), it was necessary to have access to *low Reynolds number data.* The oil data of Davis, for which R is as low as 0·14, was used here for this purpose. Thus, the water experiments performed during the present study (high R) and the oil experiments of Davis (low R) constitute the primary sources of data used here for correlation purposes. The experiments of Piret and Fand served as secondary sources of data; these secondary sources were used for decision-making purposes in cases where the correlation analysis based upon the primary sources alone yielded more than one equally good choice of values of the parameters in equation (29).

The method used here to determine the best values of the constants in equation (29) consisted of two phases: In the first phase, various combinations of values were assigned to the parameters (x_1, x_2, x_3, n, r, p and the C's) based upon previously published information; this phase culminated in the drawing of certain general conclusions. In the second phase, appropriate ranges of the parameters were investigated in detail and the final "best" values were identified. Altogether, approximately 20000 combinations of values of the parameters were evaluated. Limitations in space do not allow the inclusion here of a description of every step taken; hence, the discussion presented here contains a brief outline of the procedure used to determine the best values of the parameters in equation (29).

The general conclusions reached upon the completion of the first phase of the correlation analysis for both the film temperature correlation and the exponent correction correlation are as follows:

1. The best value for x_1 is 0·50.
2. The values of x_2 and x_3 are equal.
3. The value of C_0 is zero.

Application of these three conclusions to equation (29) results in the following simple relation:

$$N = (C_1 + C_4 R^{0 \cdot 50}) P^{x_2} \qquad (30)$$

where $C_4 = C_2 + C_3$. This equation is identical in form to McAdams' equation.

The first of the foregoing three conclusions is clearly substantiated by the results of the water experiments performed in the present study. The second and third conclusions are based primarily on Davis' oil data. These oil data exhibit considerable scatter and, for this reason, they lead to certain ambiguities. Thus, it was found that certain *unequal* pairs of values of x_2 and x_3 existed with which the data could be represented just as well as for $x_2 = x_3$; similarly, certain *non-zero* values of C_0 could be found ($C_0 \sim 0{\cdot}1$) with which the data could be represented just as well as for $C_0 = 0$. The rule adopted here for making choices in such cases was that when two correlations represent the data equally well, the algebraically simpler form is preferable. The application of this rule led to the conclusions that $x_2 = x_3$ and $C_0 = 0$; however, this is not meant to imply that these two conclusions are absolutely correct—it means only that no *better* choices can be made on the basis of presently available data.

Mean film temperature correlation

Equation (30) may be written explicitly in terms of the mean film temperature as follows:

$$N_f = (C_1 + C_4 R_f^{0 \cdot 50}) P_f^{x_2}. \qquad (31)$$

The technique used to determine the best numerical values of x_2, C_1 and C_4 (phase two of the correlation procedure) is as follows: First, an initial value of C_1 was assumed (in the vicinity of 0·3). Second, a sequence of promising values of x_2 was identified (between 0·25 and 0·35). Third, corresponding to each value of x_2 and the assumed initial value of C_1, a value of C_4 was calculated such as to yield a correlation equation having zero mean error when tested against the water data obtained in the present study. The value of C_4 so determined was used to redetermine the value of C_1 such as to yield an equation having zero mean error for the oil data of Davis. The foregoing process was repeated until a pair of values for C_1 and C_4 were found for each value of x_2 that yielded an equation having zero mean error for both the water and oil data. (In practice only one iteration was necessary.) Fourth and last, that set of values of x_2, C_1 and C_4 was adopted which yielded nearly zero mean error for Piret's and Fand's data and simultaneously yielded the *minimum standard deviation* of the error for both Davis' oil data and the water data collected during the present investigation. The final mean film temperature correlation determined by this procedure is:

$$N_f = (0{\cdot}255 + 0{\cdot}699\, R_f^{0{\cdot}50})\, P_f^{0{\cdot}29}. \quad (32)$$

The mean errors and standard deviation of the errors incurred by this equation, when tested against various sets of experimental data, are listed in Table 2. The non-zero errors which are listed in the second and fourth columns of the table for equation (32) result from rounding off the values of C_1 and C_4 to three significant figures. It is interesting to note that the term containing the Reynolds number in equation (32) is nearly identical to the analogous term in Kreith's equation for spheres, equation (21).

Calculations identical to those described above were performed for various values of j between zero and unity (see equation 2). It was verified that the optimum value of j is $\frac{1}{2}$.

Correction exponent correlation

Equation (30) may be written explicitly in terms of correction exponents as follows:

$$N_{scn} = (C_1 + C_4\, R_{scr}^{0{\cdot}50})\, P_{scp}^{x_2}. \quad (33)$$

The technique used to determine the best numerical values of x_2, n, r, p, C_1 and C_4 (phase two of the correlation procedure) is basically identical to that described above for the mean film temperature correlation. The only difference here is that, in addition to choosing a sequence of values for x_2 in the second step, appropriate sets of values of n, r and p were also chosen for evaluation. In other words, the correction exponent correlation required the evaluation of an appropriate *matrix* of values of x_2, n, r and p. The final correction exponent correlation equation determined by this procedure is:

$$N_{scn=0{\cdot}50} = (0{\cdot}221 + 0{\cdot}693\, R_{scr=0{\cdot}75}^{0{\cdot}50})\, P_b^{0{\cdot}29}. \quad (34)$$

The mean errors and standard deviation of the errors incurred by this equation, when tested against various sets of experimental data, are listed in Table 2. The non-zero errors which are listed in the second and fourth columns of the table for equation (34) result from rounding off the values of C_1 and C_4 to three significant figures. A graph of equation (34) plus the experimental data from which this equation is derived are shown in Fig. 9.

DISCUSSION OF CORRELATION EQUATIONS FOR CYLINDERS IN CROSSFLOW

The entries in Table 2 indicate that McAdams' equation correlates the data considered here better than other published equations discussed in the review of the literature [equations (15) and (16)]. Further, the table shows that equation (32) correlates the water data of the present study and Davis' oil data (primary sources) better than does McAdams' equation; and it correlates Fand's data approximately as well as does McAdams' equation. Also, the mean error of equation (32) relative to Piret's data is slightly less (better) than McAdams' equation, but the standard deviation of the error is slightly greater (worse). This worsening is due to the fact pointed out previously that Davis' and Piret's data exhibit different trends; consequently, an equation which represents Davis' data better is likely to exhibit a greater standard deviation of error with respect to Piret's data. Table 2 shows that the exponent correction method of accounting for property variation, via equation (34), is capable of representing the primary sources of data slightly more accurately than can be achieved with the mean film temperature method, via equation (32). Equations (32) and (34) both represent the secondary

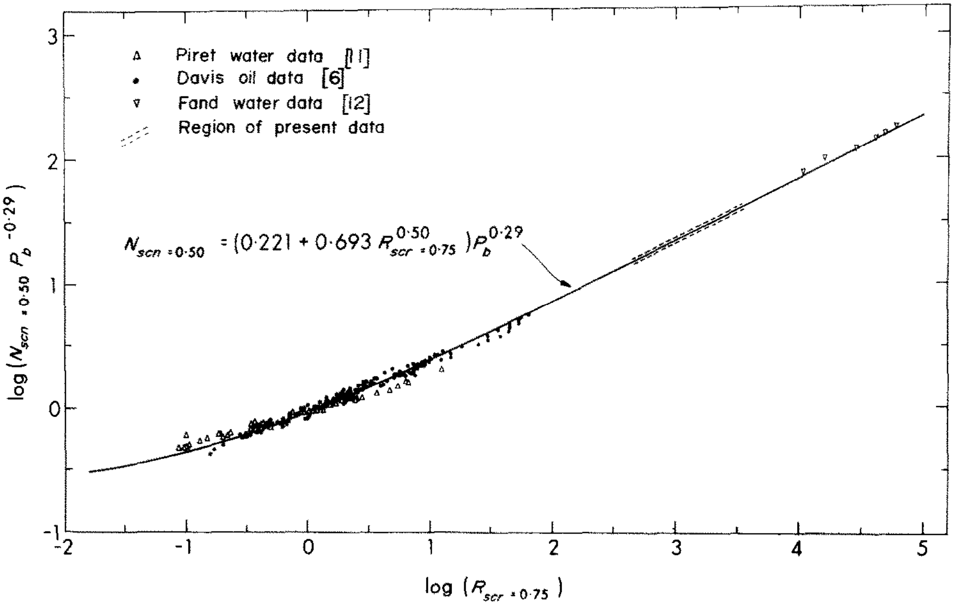

FIG. 9. Graph of equation (34) plus experimental data.

sources of data (Piret and Fand) with about the same accuracy.

The preceding observations lead to the following three conclusions worth mentioning relative to the correlation of heat transfer by forced convection from cylinders to liquids in crossflow:

1. N is a function of $R^{0.50}$ in the range $0.1 < R < 6 \times 10^4$.
2. Equation (32), which employs the mean film temperature method of accounting for property variation, correlates the experimental data considered here better than any of the equations discussed in the review of the literature. Equation (32) provides results that are sufficiently accurate for most practical purposes.
3. Equation (34), which employs the correction exponent method of accounting for property variation, correlates the experimental data considered here with slightly greater accuracy than does equation (32). This fact demonstrates the applicability of the correction exponent method.

CORRELATIONS FOR GEOMETRIES OTHER THAN CYLINDERS

Spheres in forced flow

Equation (18) correlates the experimental data of Vliet and Leppert [3] for spheres with a mean error of -2.55 per cent and standard deviation of the error equal to 2.16 per cent. The following equation, which employs the exponent correction method of accounting for property variation, correlates Vliet and Leppert's data with zero mean error and standard deviation of the error equal to 2.69 per cent:

$$N_{scn=0.50} = (2.70 + 0.126\, R_{scr=0.75}^{0.66})\, P_b^{0.5}. \quad (35)$$

Equation (19) correlates the data of Vliet and Leppert with a mean error of -1.90 per cent and standard deviation of the error equal to 7.42 per cent. The following equation, which employs the exponent correction method of accounting for property variation, correlates Vliet and Leppert's data with zero mean error and standard deviation of the error equal to 7.50 per cent:

$$N_{scn=0.50} = (1.20 + 0.558\, R_{scr=0.75}^{0.54})\, P_b^{0.3}. \quad (36)$$

Thus, equations (35) and (36) represent Vliet and Leppert's data with equal or better accuracy than do equations (18) and (19).* The values of the correction exponents in equations (35) and (36)—$n = 0{\cdot}50$, $r = 0{\cdot}75$ and $p = 1$—are identical to those determined in the correlation analysis for cylinders. It is inferred, therefore, that this set of values ($n = 0{\cdot}50$, $r = 0{\cdot}75$ and $p = 1$) is universally applicable to heat transfer from blunt bodies such as is exemplified by cylinders and spheres.

Turbulent flow in pipes

When Petukhov's equation, equation (27), was tested against the data of Kreith and Summerfield [19] (heating of n-butyl alcohol) and of Morris and Whitman [15] (heating of water and straw oil) it was found that, for $R > 10000$ (turbulent flow), best results were obtained when i was taken equal to 0·06. Table 3 shows the errors incurred by using equation (27) with $i = 0{\cdot}06$. The value $i = 0{\cdot}06$ determined here is nearly equal to the value $i = 0{\cdot}05$ suggested by Malina and Sparrow.

The correction exponent method of accounting for property variation was applied to turbulent pipe flow by replacing N_{0b}, R_b and P_b in equation (27) by N_{scn}, R_{scr} and P_{scp} and then determining a set of values of n, r and p such that the resulting equation would correlate the data of Kreith and Summerfield and of Morris and Whitman. The set of values so determined is $n = p = 1$ and $r = 0{\cdot}92$, with which equation (27) becomes:

$$N_b = \frac{(\xi/8)\, R_{scr=0\cdot92}\, P_b}{K_1 + K_2\,(\xi/8)^{\frac{1}{2}}\,(P_b^{\frac{2}{3}} - 1)} \qquad (37)$$

where $\xi = (1{\cdot}82 \log R_{scr=0\cdot92} - 1{\cdot}64)^{-2}$, $K_1 = 1 + 3{\cdot}4\ \xi$ and $K_2 = 11{\cdot}7 + 1{\cdot}8\ P_b^{-\frac{1}{3}}$. Table 3 shows the errors incurred by using equation (37). The magnitudes of these errors are not significantly different from the errors incurred by using equation (27) with $i = 0{\cdot}06$. It is concluded, therefore, that the correction exponent method of accounting for property variation is applicable to turbulent flow in pipes and that $n = p = 1$, $r = 0{\cdot}92$ is an appropriate set of values of the correction exponents.

Table 3. Errors incurred by different correlation equations for turbulent pipe flow when tested against two sets of experimental data

Equation	Morris and Whitman Data [15]		Kreith and Summerfield Data [19]	
	Mean error, %	Standard deviation of error, %	Mean error, %	Standard deviation or error, %
Equation (27) with $i = 0{\cdot}06$	0·45	3·87	−0·64	3·83
Equation (37)	0·65	3·89	0·16	3·82

It has been demonstrated that the correction exponent for the Prandtl number for heat transfer from cylinders, spheres and in turbulent pipe flow is unity (*nota bene*, $P_{scp=1} = P_b$). This consistency suggests that in *all* cases of heat transfer to liquids by forced convection, the Nusselt number is a function of P_b. It is instructive to note that, if n and p are both taken equal to unity and i is taken to be 0·06, the value of the correction exponent of the Reynolds number for turbulent pipe flow, $r = 0{\cdot}92$, can be very nearly deduced for liquids from equation (25). Thus, if $i = 0{\cdot}06$ instead of 0·14, equation (25) may be written as follows:

$$N_b = 0{\cdot}027 \left[R_b \left(\frac{\mu_b}{\mu_s} \right)^{0\cdot075} \right]^{0\cdot8} P_b^{\frac{1}{3}}. \qquad (39)$$

If the quantity in square brackets is identified with R_{scr} it follows that

$$R_{scr} = R_b \left(\frac{\mu_b}{\mu_s} \right)^{0\cdot075}. \qquad (40)$$

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* This statement does not imply that the data themselves are accurate.

For constant density fluids (liquids) the preceding equation reduces to:

$$\left(\frac{\mu_b}{\mu_s}\right)^{1-r} = \left(\frac{\mu_b}{\mu_s}\right)^{0\cdot075} \qquad (41)$$

from which $r = 0{\cdot}925$—nearly 0·92. The quantity $R_{scr=0\cdot92}$ may be regarded as the Reynolds number which characterizes the flow with respect to heat transfer for turbulent flow in pipes. Equation (37) requires the performance of tedious calculations; hence, to facilitate its application to design, this equation is plotted in Fig. 10.

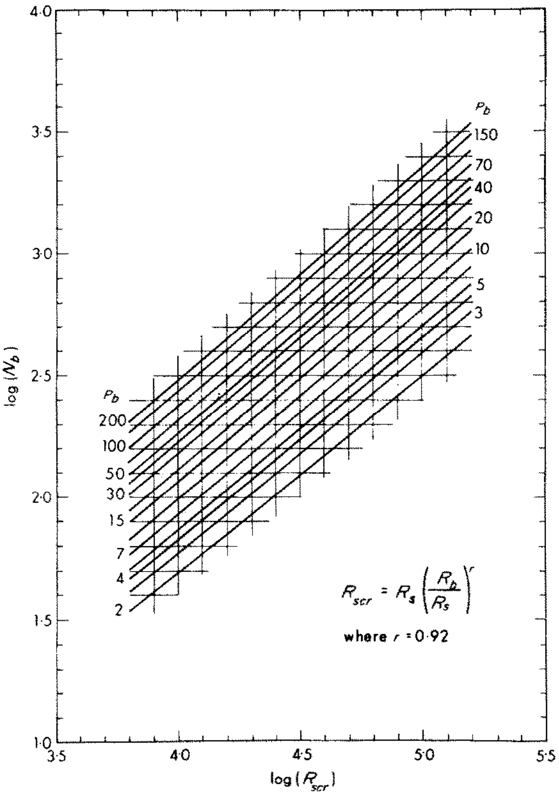

FIG. 10. Graph of equation (37).

RESUME AND CONCLUSIONS

The results of the present study show that heat transfer by forced convection from cylinders to liquids in crossflow can be represented accurately by either of the two following equations:

$$N_f = (0{\cdot}255 + 0{\cdot}699\, R_f^{0\cdot50})\, P_f^{0\cdot29} \qquad (32)$$

$$N_{scn=0\cdot50} = (0{\cdot}221 + 0{\cdot}693\, R^{0\cdot50}_{scr=0\cdot75})\, P_b^{0\cdot29}. \qquad (34)$$

Equation (34) demonstrates the applicability of the correction exponent method of accounting for property variation. It has been shown that the set of values of the correction exponents determined on the basis of experimental data gathered for cylinders—$n = 0{\cdot}50$, $r = 0{\cdot}75$, $p = 1$—can also be used to correlate heat transfer by forced convection from spheres. It is concluded, therefore, that this set of values of n, r and p is universally applicable for heat transfer from bluff bodies to external flows of liquids.

It has been shown that the correction exponent method of accounting for property variation is applicable to the heating of liquids flowing turbulently in pipes. The set of values $n = p = 1$, $r = 0{\cdot}92$ accurately accounts for property variation for this geometry when it is used in conjunction with Petukhov's analysis, per equation (37).

The correction exponent method of accounting for property variation eliminates the need to use empirical correction factors, such as $(\mu_b/\mu_s)^i$, which are not derivable from the usual dimensional analysis nor from the governing set of differential equations. On the basis of the good results obtained here in conjunction with Petukhov's mathematical analysis of heat transfer in turbulent pipe flow, it is suggested that analytical predictions of heat transfer in the case of variable property fluids might generally be obtained by first solving the associated constant property problems and then simply replacing the Nusselt, Reynolds and Prandtl numbers where they appear in the constant property solutions by N_{scn}, R_{scr} and P_{scp} with an appropriate set of numerical values of the correction exponents n, r and p. Although the correction exponent method of accounting for property variation has been applied here exclusively to liquids, there is no reason why it cannot be also applied to gases.

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INFLUENCE DE LA VARIATION DES PROPRIÉTÉS SUR LE TRANSFERT THERMIQUE CONVECTION FORCÉE POUR LES LIQUIDES

Résumé—Dans cet article on présente une nouvelle méthode pour rendre compte de l'influence de la variation des propriétés sur le transfert thermique en convection forcée entre des surfaces chauffées et des liquides. La méthode est utilisée en liaison avec des nouveaux résultats expérimentaux pour développer une équation représentant le transfert thermique par convection forcée entre des cylindres et des liquides en écoulement frontal. La méthode est encore appliquée au transfert thermique par convection forcée à partir de sphères dans des liquides, ainsi qu'à des écoulements turbulents de liquides dans des tuyaux.

DER EINFLUSS VERÄNDERLICHER STOFFWERTE AUF DEN WÄRMEÜBERGANG BEI ERZWUNGENER KONVEKTION IN FLÜSSIGKEITEN

Zusammenfassung—In diesem Beitrag wird eine neue Methode vorgestellt, die den Einfluss der Stoffwertänderungen auf den Wärmeübergang von beheizten Oberflächen auf Flüssigkeiten berücksichtigt. Diese Methode wird benützt im Zusammenhang mit neuen Versuchsdaten, um eine Beziehungsgleichung für den Wärmeübergang von querangeströmten Zylindern an Flüssigkeiten bei Zwangskonvektion zu entwickeln. Die Methode wird auch angewandt für den Wärmeübergang an Kugeln und der turbulenten Rohrströmung.

ВЛИЯНИЕ ИЗМЕНЯЕМОСТИ ФИЗИЧЕСКИХ СВОЙСТВ ЖИДКОСТИ НА ТЕПЛОПЕРЕНОС ПРИ ВЫНУЖДЕННОЙ КОНВЕКЦИИ

Аннотация—В этой статье описывается новый метод расчёта влияния изменяемости свойств жидкости на конвективный перенос тепла от нагретых поверхностей. Метод предусматривает использование новых экспериментальных данных для получения замкнутого корреляционного уравнения конвективного переноса тепла от цилиндров, поперечно обтекаемых жидкостью. Этот метод также используется для расчета конвективного переноса тепла от сфер к жидкостям и турбулентного течения жидкостей в трубах.